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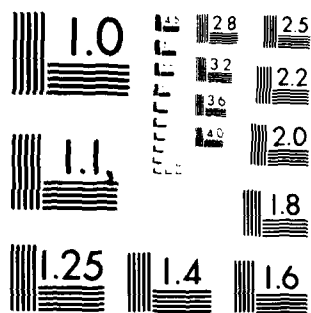
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ON DIFFERENTIAL EQUATIONS OF NONLOCAL ELASTICITY  
AND  
SOLUTIONS OF SCREW DISLOCATION AND SURFACE WAVES

A. Cemal Eringen  
PRINCETON UNIVERSITY

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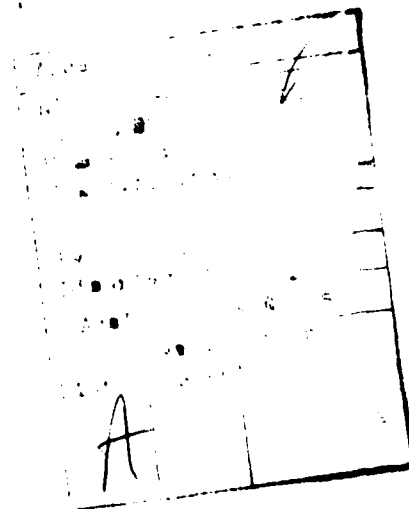
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ABSTRACT

Integro-partial differential equations of the linear theory of nonlocal elasticity are reduced to singular partial differential equations for a special class of physically admissible kernels. Solutions are obtained for the screw dislocation and surface waves. Experimental observations and atomic lattice dynamics appear to support the theoretical results very nicely.



## 1. INTRODUCTION

In the theory of nonlocal elasticity<sup>1,2</sup> the stress at a reference point  $\underline{x}$  is considered to be a functional of the strain field at every point  $\underline{x}'$  in the body. For homogeneous, isotropic bodies, the linear theory leads to a set of integro-partial differential equations for the displacement field, which are generally difficult to solve. For a spacial class of kernels, these equations are reducible to a set of singular partial differential equations for which the literature is extensive.

The selection of the appropriate class of kernels is not ad hoc but fairly general, based on mathematical conditions of admissibility and physical conditions of verifiability. For example, the dispersion curves available from lattice dynamics and phonon dispersion experiments provide excellent testing on the success of these kernels. Ultimately, these kernels should be expressed in terms of interatomic force potentials or correlation functions. Presently, several solutions obtained for various problems support the theory advanced here. For example, the dispersion curve,<sup>1,3</sup> obtained for plane waves are in excellent agreement with those of the Born-Kármán theory of lattice dynamics. The dislocation core and cohesive (theoretical) stress predicted by nonlocal theory<sup>4,5</sup> are close to those known in the physics of solids. Moreover, nonlocal theory reduces to classical (local theory) in the long wave-length limit and to atomic lattice dynamics in the short wave-length limit.<sup>6</sup> These and several other considerations lead us to the exciting prospect that by means of nonlocal elasticity, excellent approximation may be provided for a large class of physical phenomena with characteristic lengths ranging from microscopic to macroscopic scales. This situation becomes specially promising in dealing with imperfect solids, dislocations and fracture, since in these cases, the internal (atomic) state of the body is difficult to characterize.

The solution of nonlocal elasticity problems are however difficult mathematically, since little is known on the treatment of integro-partial differential equations, especially for mixed boundary value problems. Therefore, the treatment of these problems by means of singular differential equations is promising. This is born out, at least, with the treatment of two problems here, namely the screw dislocation and Rayleigh surface waves.

Results for both problems are gratifying in that they are supported by atomic lattice dynamics and experiments.

The formulation of nonlocal elasticity in terms of singular differential equations is reminiscent of other fields of physics and it brings unification of various methodology. These are indicated briefly in Section 3.

## 2. NONLOCAL ELASTICITY

In several previous papers, we developed a theory of nonlocal elasticity, cf. [1 - 4]. According to this theory, the stress at a reference point  $\underline{x}$  in the body depends not only on the strains at  $\underline{x}$  but also on strains at all other points of the body. This observation is in accordance with atomic theory of lattice dynamics and experimental observations on phonon dispersion. In the limit when the effects of strains at points other than  $\underline{x}$  is neglected, one obtains classical (local) theory of elasticity.

For homogeneous and isotropic elastic solids, the linear theory is expressed by the set of equations

$$(2.1) \quad t_{k\ell,k} + \rho(f_\ell - \ddot{u}_\ell) = 0$$

$$(2.2) \quad t_{k\ell}(\underline{x}) = \int_V \alpha(|\underline{x}' - \underline{x}|, \tau) \sigma_{k\ell}(\underline{x}') dv(\underline{x}')$$

$$(2.3) \quad \sigma_{k\ell}(\underline{x}') = \lambda e_{rr}(\underline{x}') \delta_{k\ell} + 2\mu e_{k\ell}(\underline{x}')$$



$$(2.4) \quad e_{k\ell}(\underline{x}') = \frac{1}{2} \left[ \frac{\partial u_k(\underline{x}')}{\partial x'_\ell} + \frac{\partial u_\ell(\underline{x}')}{\partial x'_k} \right]$$

where  $t_{k\ell}$ ,  $\rho$ ,  $f_\ell$  and  $u_\ell$  are respectively, the stress tensor, mass density, body force density and the displacement vector at a reference point  $\underline{x}$  in the body, at time  $t$ .  $\sigma_{k\ell}(\underline{x}')$  is the macroscopic (classical) stress tensor at  $\underline{x}'$  which is related to the linear strain tensor  $e_{k\ell}(\underline{x}')$  at any point  $\underline{x}'$  in the body at time  $t$ , with  $\lambda$  and  $\mu$  being Lamé constants. The only difference between (2.1) to (2.4) and the corresponding equations of classical elasticity is in the constitutive equations (2.2) which replaces Hooke's law (2.3) by (2.2). The volume integral in (2.2) is over the region  $V$  occupied by the body.

Field equations of nonlocal elasticity are obtained by combining (2.1) to (2.4). We substitute Eq. (2.2) into (2.1) and use the identity

$$\begin{aligned} \frac{\partial \alpha}{\partial x'_k} \sigma_{k\ell}(\underline{x}') &= - \frac{\partial \alpha}{\partial x'_k} \sigma_{k\ell}(\underline{x}') \\ &= - \frac{\partial}{\partial x'_k} [\alpha \sigma_{k\ell}(\underline{x}')] + \alpha \frac{\partial \sigma_{k\ell}}{\partial x'_k} \end{aligned}$$

to obtain

$$(2.5) \quad - \int_{\partial V} \alpha(|\underline{x}' - \underline{x}|) \sigma_{k\ell}(\underline{x}') n'_k da(\underline{x}') + \int_V \alpha(|\underline{x}' - \underline{x}|) \sigma_{k\ell,k'} dv(\underline{x}') + \rho(f_\ell - \ddot{u}_\ell) = 0$$

Here the first integral, over the surface of the body, represents the surface stresses, (e.g. surface tension). Consequently, nonlocal theory accounts for

surface physics, an important asset not included in classical theories. Substitution of Eqs. (2.3 and 2.4) into (2.5) gives the field equations

$$(2.6) \quad - \int_{\partial V} \alpha(|\underline{x}' - \underline{x}|) [\lambda u'_{r,r} \delta_{k\ell} + \mu(u'_{k,\ell} + u'_{\ell,k})] n'_k da' \\ + \int_V \alpha(|\underline{x}' - \underline{x}|) [(\lambda + \mu) u'_{k,\ell k} + \mu u'_{\ell, kk}] dv' + \rho(f_\ell - \ddot{u}_\ell) = 0$$

where a prime (') denotes dependence on  $\underline{x}'$ , e.g.  $u' = u(\underline{x}')$ .

Integral equations (2.6) must be solved to determine the displacement field  $u(\underline{x}, t)$  under appropriate boundary and initial conditions. Boundary and initial conditions involving the displacement and velocity fields are identical to those of the classical theory. Boundary conditions involving tractions is based on the stress tensor  $t_{k\ell}$ , not on  $\sigma_{k\ell}$ , i.e.

$$(2.7) \quad t_{k\ell} n_k = t_{(\eta)\ell}$$

where  $t_{(\eta)\ell}$  are the prescribed boundary tractions.

### 3. DETERMINATION OF NONLOCAL MODULUS

From the structure of the constitutive equations (2.2), it is clear that the *nonlocal modulus*  $\alpha(|\underline{x}' - \underline{x}|)$  has the dimension of  $(\text{length})^{-3}$ . Therefore, it will depend on a characteristic length ratio  $a/\ell$ , where  $a$  is an internal characteristic length (e.g. lattice parameter, granular

distance) and  $\ell$  is an external characteristic length (e.g. crack length, wave length). We may express  $\alpha$  in a more appropriate form as

$$(3.1) \quad \alpha = \alpha(|\underline{x}' - \underline{x}|, \tau), \quad \tau = e_0 a / \ell$$

where  $e_0$  is a constant appropriate to each material.

The nonlocal modulus has the following interesting properties.

- (i) It acquires its maximum at  $\underline{x}' = \underline{x}$  attenuating with  $|\underline{x}' - \underline{x}|$ .
- (ii) When  $\tau \rightarrow 0$ ,  $\alpha$  must revert to the Dirac delta measure so that classical elasticity limit is included in the limit of vanishing internal characteristic length.

$$(3.2) \quad \lim_{\tau \rightarrow 0} \alpha(|\underline{x}' - \underline{x}|, \tau) = \delta(|\underline{x}' - \underline{x}|)$$

We therefore expect that  $\alpha$  is a delta sequence.

- (iii) For small internal characteristic lengths, i.e. when  $\tau \rightarrow 1$ , nonlocal theory should approximate atomic lattice dynamics.

In fact, by discretizing Eq. (2.2), it can be shown that equations of nonlocal elasticity reverts to those of atomic lattice dynamics<sup>6</sup>.

- (iv) By matching the dispersion curves of plane waves with those of atomic lattice dynamics (or experiments), we can determine  $\alpha$  for a given material. Several different forms have been found<sup>1,3,7</sup>.

The following are some examples which have found applications:

(a) One-Dimensional Moduli

$$(3.3) \quad \alpha(|x|, \tau) = \frac{1}{\ell\tau} \left(1 - \frac{|x|}{\ell\tau}\right), \quad |x| \leq \ell\tau$$

$$= 0, \quad |x| \geq \ell\tau$$

$$(3.4) \quad \alpha(|x|, \tau) = \frac{1}{2\ell\tau} e^{-|x|/\ell\tau}$$

$$(3.5) \quad \alpha(|x|, \tau) = \frac{1}{\ell\sqrt{\pi\tau}} \exp(-x^2/\ell^2\tau)$$

(b) Two-Dimensional Moduli

$$(3.6) \quad \alpha(|\underline{x}|, \tau) = (2\pi\ell^2\tau)^{-1} K_0(\sqrt{\underline{x} \cdot \underline{x}}/\ell\tau)$$

where  $K_0$  is the modified Bessel function

$$(3.7) \quad \alpha(|\underline{x}|, \tau) = (\pi\ell^2)^{-1} \exp(-\underline{x} \cdot \underline{x} / \ell^2\tau)$$

(c) Three-Dimensional Moduli

$$(3.8) \quad \alpha(|\underline{x}|, t) = \frac{1}{8(\pi t)^{3/2}} \exp(-\underline{x} \cdot \underline{x} / 4t), \quad t = \ell^2\tau/4$$

$$(3.9) \quad \alpha(|\underline{x}|, \tau) = (4\pi\ell^2\tau)^{-1} (\underline{x} \cdot \underline{x})^{-1/2} \exp(-\sqrt{\underline{x} \cdot \underline{x}} / \tau\ell)$$

We note that Eq. (3.3) gives a perfect match of the dispersion curve of one-dimensional plane waves based on the nonlocal elasticity and the Born-Kármán model of the atomic lattice dynamics<sup>3</sup>. For two-dimensional lattices, Eq. (3.6) provides an excellent match with atomic dispersion curves, with a maximum error less than 1.2%, Ari and Eringen<sup>7</sup>. All other nonlocal moduli also provide excellent approximation to the atomic dispersion curves, for a choice of  $e_0$ , Eringen<sup>8</sup>.

(v) We observe that all nonlocal moduli given above are realized so that their integrals over the domain of integration (line, surface, volume) give unity. Moreover, they are all  $\delta$ -sequence, i.e. when  $\tau \rightarrow 0$  we obtain the Dirac delta function. Because of this property, nonlocal elasticity in the limit  $\tau \rightarrow 0$  reverts to classical elasticity as can be seen by letting  $\tau \rightarrow 0$  in (2.2), to obtain Hookes law of classical elasticity.

We now exploit this observation further by assuming that:

(vi)  $\alpha$  is Green's function of a linear differential operator:

$$(3.10) \quad L \alpha(|x' - x|, \tau) = \delta(|x' - x|)$$

If such an operator can be found, then applying  $L$  to (2.2), we obtain

$$(3.11) \quad L t_{k\ell} = \sigma_{k\ell}$$

In particular, if  $L$  is a differential operator with constant coefficients, then

$$(3.12) \quad (L t_{k\ell})_{,k} = L t_{k\ell,k}$$

and (2.1) gives

$$(3.13) \quad \sigma_{k\ell,k} + L(\rho f_{\ell} - \rho \ddot{u}_{\ell}) = 0$$

In this case, we have partial differential equations to solve, instead of integro-partial differential equations.<sup>9</sup> This, of course, provides a great deal of simplicity over the original equations (2.6). In particular, for static problems with vanishing body forces (or more generally, when  $L(\rho f_{\ell} - \rho \ddot{u}_{\ell}) = 0$ ), we have the classical equations of equilibrium

$$(3.14) \quad \sigma_{k\ell,k} = 0$$

which upon using (2.3) gives Naviers' equations.

The nonlocal modulus (3.8) is a Green's function which satisfies the differential equation

$$(3.15) \quad \nabla^2 \alpha - \frac{\partial \alpha}{\partial t} = 0, \quad \alpha(\underline{x}, 0) = \delta(\underline{x})$$

The fact that this is the case is well-known since the solution  $\alpha(\underline{x}, t)$  of the diffusion equation

$$(3.16) \quad \nabla^2 \underline{t} - \frac{\partial \underline{t}}{\partial \tau} = \underline{0}$$

which, for  $t=0$  coincide with a given continuous function  $\underline{g}(\underline{x})$ , is given by

$$(3.17) \quad \underline{t} = \int_V \alpha(|\underline{x}' - \underline{x}|, \tau) \phi(\underline{x}') dv(\underline{x}')$$

when  $V$  extends to infinity in all directions, cf. Courant.<sup>10</sup>

Eq. (3.17) is valid even when  $V$  is finite and the reference point  $\underline{x}$  is not too close to the boundaries, since  $\alpha$  attenuates rapidly to zero with  $|\underline{x}' - \underline{x}|$ .

Of course the differential operator  $L$  may be different than the diffusion operator. For example, for Eq. (3.6), one can see that

$$(3.18) \quad L = 1 - \tau^2 \nabla^2 \nabla^2$$

i.e., we have

$$(3.19) \quad (1 - \tau^2 \nabla^2 \nabla^2) \underline{t} = \underline{g}$$

In fact, this result can be justified by an approximation of the atomic dispersion relations. To see this, consider the Born-Kármán model of lattice dynamics and equate the expression of the frequency given there to that of nonlocal theory for plane waves.

$$(3.20) \quad \bar{\alpha}(k) = (\omega/\omega_0)^2 = (2/k^2 a^2)(1 - \cos ka)$$

We rewrite this expression in the form

$$(3.21) \quad 2(1 - \cos ka)/k^2 a^2 = 1 - \frac{k^2 a^2}{12} + \frac{k^4 a^4}{360} - \dots$$

$$\approx (1 + e_0^2 a^2 k^2)^{-1}$$

for  $k^2 a^2 \ll 1$ . This is permissible since according to Tauberian theorems, the behavior of a function near infinity is reflected in the behavior of its Fourier transform near the origin. Thus, this approximation is tantamount to approximating  $\alpha(|x|)$  for large  $|x|$ . In fact, by this process, the boundary of the Brillouin zone is thrown to infinity so that  $\alpha$  does not have a finite support. By matching  $\bar{\alpha} = (1 + e_0^2 a^2 k^2)^{-1}$  obtained this way with Eq. (3.20) at the end of the Brillouin zone, we obtain a curve which approximates the atomic dispersion curve quite well.

In Figure 1, the comparison is made for this matching, i.e.

$$(3.22) \quad \begin{aligned} \omega a/c_0 &= k a (1 + e_0^2 a^2 k^2)^{-1/2} && \text{(Nonlocal)} \\ \omega a/c_0 &= 2 \sin(ka/2) && \text{(Lattice Dynamics)} \end{aligned}$$

The matching is perfect at  $ka = \pi$ , which leads to

$$(3.23) \quad e_0 \approx 0.39$$

The maximum error is of the order of 6% in  $|ka| < \pi$ . Note, however, that the group velocity at  $ka = \pi$  is badly off and this is the price we have to pay.



If we accept this approximation, Eq. (3.21) upon inversion, gives

$$(3.24) \quad (1 - \tau_l^2 \nabla^2)_\alpha = \delta(|\underline{x}|) ; \quad \tau_l \equiv e_0 a$$

The application of this operator to Eq. (3.17) then, leads to (3.19). Of course, other types of approximations are possible.

It is not difficult to determine  $L$  for other moduli. In fact, for Gaussian kernels, (3.5) and (3.7) are similar to (3.15).

(vii) The above considerations further suggest that Eq. (3.17) may be considered as the probabilistic average of  $\underline{\sigma}$ , if  $\alpha$  is considered to be a probability density function. In fact, (3.8) is non-other than the Gaussian density function. Such a consideration resembles the method of analysis of quantum mechanics with the probability density function satisfying a diffusion equation.

(viii) A somewhat different interpretation of (3.16) is made by considering  $\alpha$  as a correlation function, in which case, it should be possible to determine  $\alpha$  from statistical mechanical considerations. Such a study is now underway.

#### 4. SCREW DISLOCATION REVISITED

Consider a screw dislocation, in the sense of Volterra, located at the plane  $x_3=0$  of rectangular coordinates  $x_k$ , Fig. 2. The displacement field has only single component  $u_3(x_1, x_2, t)$ . The stress field is determined by solving Eq. (3.16) whose Laplace transform with respect to  $t$  is

$$(4.1) \quad \nabla^2 \bar{t}_{k\ell} - s \bar{t}_{k\ell} = \sigma_{k\ell}$$

where  $\sigma_{k\ell}$  has only two non-vanishing components

$$(4.2) \quad \sigma_{31} = \mu \frac{\partial u_3}{\partial x_1}, \quad \sigma_{32} = \mu \frac{\partial u_3}{\partial x_2}$$

In (4.1), a superposed bar indicates the Laplace transform and  $s$  is the transform variable.

The divergence of Eq. (4.1), upon using (2.1) and (4.2), gives

$$(4.3) \quad \sigma_{k\ell,k} = (\nabla^2 - s) \rho \ddot{u}_\ell$$

For the static case, Eq. (4.3) gives

$$(4.4) \quad \nabla^2 u_3 = 0$$

whose solution is

$$(4.5) \quad u_3 = \frac{b}{2\pi} \tan^{-1}(x_2/x_1)$$

where  $b$  is the Burger's vector. In polar coordinates  $(r, \theta, z)$  this is equivalent to

$$(4.6) \quad u_3 = \frac{b}{2\pi} \theta$$

$$(4.7) \quad \sigma_{31} = -\frac{\mu b}{2\pi r} \sin \theta, \quad \sigma_{32} = \frac{\mu b}{2\pi r} \cos \theta$$

Carrying (4.7) into (4.1), we obtain differential equations for  $\bar{t}_{31}$  and  $\bar{t}_{32}$ , whose general solutions, having proper symmetry with respect to  $\pm \theta$ , are given by

$$(4.8) \quad \bar{t}_{31} = \bar{T}_1(\rho) \sin \theta, \quad \bar{t}_{32} = \bar{T}_2(\rho) \cos \theta,$$

where

$$(4.9) \quad \bar{T}_\alpha(\rho) = A_\alpha K_1(\rho) + B_\alpha I_1(\rho) + (-)^\alpha \frac{C}{\rho}, \quad \alpha = 1, 2.$$

$$(4.10) \quad \rho = \sqrt{s} r, \quad C = \mu b / 2\pi \sqrt{s}$$

Here,  $I_1(\rho)$  and  $K_1(\rho)$  are modified Bessels' functions and  $A_\alpha$  and  $B_\alpha$  are arbitrary constants. The stress field must vanish as  $\rho \rightarrow \infty$ . This implies that  $B_\alpha = 0$ . In cylindrical coordinates  $(r, \theta, z)$ , the stress field is given by

$$(4.11) \quad \begin{aligned} \bar{t}_{zr} &= (A_1 + A_2) K_1(\rho) \sin \theta \cos \theta \\ \bar{t}_{z\theta} &= (-A_1 \sin^2 \theta + A_2 \cos^2 \theta) K_1(\rho) + C \rho^{-1} \end{aligned}$$

If we imagine the edge line  $\rho = 0$  of the dislocation as a limit  $\epsilon \rightarrow 0$  of a small cylindrical surface with radius  $r = \epsilon$ , then  $t_{zr}$  must vanish as  $\epsilon \rightarrow 0$ . This, through (4.11), gives  $A_2 = -A_1$  so that

$$(4.12) \quad \bar{t}_{zr} = 0, \quad \bar{t}_{z\theta} = -A_1 K_1(\rho) + C \rho^{-1}$$

The hoop stress  $t_{z\theta}$  will be regular at  $\rho=0$ , if and only if,  $A_1 = C$ .

Consequently,

$$(4.13) \quad \bar{t}_{z\theta} = \frac{\mu b}{2\pi r} \left[ \frac{1}{s} - \frac{r}{\sqrt{s}} K_1(r\sqrt{s}) \right]$$

$$(4.14) \quad t_{z\theta} = \frac{\mu b}{2\pi r} (1 - e^{-r^2/4t}) .$$

Remembering  $t = l^2 \tau / 4$  and in terms of the definition of  $\alpha$  given in Ref. [4],  $t = a^2 / 4k^2$ , this result is in agreement with that found earlier in a different way.<sup>4</sup> We observe that the displacement field (4.5) of a screw dislocation in a nonlocal elastic medium is the same as in a classical elasticity, even though stress field is different. This is the result of the particular choice of the kernel. The effects of anholonomicity of the dispersion curve near the boundaries of the Brillouin zone and the nonlinear force law are ignored.

Had we employed the operator (3.18) instead of (3.15), we would have obtained

$$(4.15) \quad t_{z\theta} = \frac{b}{2\pi r} \left[ 1 - \frac{r}{\tau l} K_1(r/\tau l) \right], \quad \text{all other } t_{kl} = 0 .$$

This is also regular for all  $r$  in the interval  $0 \leq r \leq \infty$ .

In non-dimensional form, (4.14) may be written as

$$(4.16) \quad T_\theta(\rho) = \rho^{-1} (1 - e^{-\rho^2})$$

wh

$$(4.17) \quad T_\theta = 2\pi a t_{z\theta} / \mu b k = \rho^{-1} (1 - e^{-\rho^2}), \quad \rho \equiv kr/a$$

$T_\theta(\rho)$  possesses a maximum at  $\rho_c$  which is the root of

$$(4.18) \quad 1 + 2 \rho_c^2 = e^{\rho_c^2}, \quad \rho_c = 1.1209$$

and  $t_{\theta\max}$  is given by

$$(4.19) \quad T_{\theta\max} = \frac{2 \rho_c}{1 + 2 \rho_c^2} \approx 0.6332$$

The plot of  $T_\theta(\rho)$  versus  $\rho$  is shown in Fig. 3. For a perfectly brittle crystal, the theoretical shear strength  $t_y$  is reached when  $t_{z\theta} = t_y$ . Consequently,

$$(4.20) \quad t_y = 0.6332 \frac{\mu b k}{2\pi a}$$

In a previous<sup>8</sup> paper, we have shown that  $k=1.65$  gives a perfect match in the entire Brillouin zone of Born-Kármán lattice with an error less than 0.2%. Using this value and  $b/a=1/\sqrt{2}$  for fcc materials in (4.20) we find

$$(4.21) \quad t_y/\mu = 0.12$$

This result compares well with the value 0.11 for Al (f.c.c.); W,  $\alpha$ -Fe (b.c.c.) and 0.12 for Na Cl, MgO. (cf. Lawn and Wilshaw, p. 160).

In examining Fig. 2 closely, we note that when  $t_y = t_{z\theta\max}$  a brittle perfect crystal will rupture. However, if the crystal is ductile at this point, dislocations will be produced. Thus the region around the crack tip  $0 \leq \rho < \rho_c$  is a dislocation-free zone, i.e. dislocations will emerge at  $\rho = \rho_c$  and will pile up in a region  $\rho \geq \rho_c$ . This prediction of the present theory is supported by the recent observations made in electron microscopy<sup>11</sup>.

According to the present theory, the rupture or dislocation initiation then begins not at the center of the dislocation  $\rho = 0$  but at a critical distance  $\rho = \rho_c > 0$ . This is against our previous understanding of the dislocation and fracture mechanism but supported by experiments.

Finally, we may wish to enquire whether these predictions are greatly affected by the kernel chosen. In the case of the kernel (3.6), leading to Eq. (4.15), the  $T_\theta(\rho)$ -curve is also shown in Fig. 2 with  $\rho_c$ ,  $T_{\theta\max}$  and  $t_y$  given by

$$(4.22) \quad \rho_c = 1.1, \quad T_{\theta\max} = 0.3995,$$

$$(4.23) \quad t_y = 0.3995 \frac{\mu b}{2\pi e_0 a}.$$

If we write  $h = e_0 a / 0.3995$ , Eq. (4.23) agrees with the estimate of Frenkel based on an atomic model (cf. Kelly<sup>12</sup>, p. 12). In the case of the kernel (3.6), the match of the dispersion curves were provided for the value of  $e_0 = 0.39$  with an error less than 6% at a point in the Brillouin zone (Eq. 3.23). Using this value, we again obtain  $t_y/\mu = 0.12$ . Thus, in spite of the difference in the two curves in Fig. 2, the resulting theoretical strengths are not too far off from each other. Moreover, the location of the maxima are nearly coincident so that the origin of the dislocation generation or rupture are predicted to be the same.

## 5. RAYLEIGH SURFACE WAVES

In a previous paper<sup>13</sup>, we have determined the phase velocity of Rayleigh surface waves and found that they are dispersive. In these calculations, one-dimensional kernel (3.3) was used with nonlocal effects taken only in the direction of boundary line,  $x_2 = 0$  in two-dimensional medium  $0 \leq x_2 \leq \infty$ ,  $-\infty < x_1 \leq \infty$ . Here, we take advantage of the two-dimensional kernel (3.6) which reduces constitutive equations to the form (3.19). Upon taking the divergence of (3.19) and using (2.1), (2.3) and (2.4) with  $\underline{f} = \underline{0}$ , we obtain

$$(5.1) \quad (\lambda + \mu) u_{k,k\ell} + \mu u_{\ell,kk} - (1 - \tau_{\ell}^2 \nabla^2) \ddot{u}_{\ell} = 0$$

For the plane-strain, we introduce Lamé potentials  $\phi(\underline{x}, t)$  and  $\psi(\underline{x}, t)$ , ( $\underline{x} = \{x_1, x_2\}$ ) by

$$(5.2) \quad u_1 = \frac{\partial \phi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}, \quad u_2 = \frac{\partial \phi}{\partial x_2} - \frac{\partial \psi}{\partial x_1}$$

leading to

$$(5.3) \quad \begin{aligned} c_1^2 \nabla^2 \phi - (1 - \tau_{\ell}^2 \nabla^2) \ddot{\phi} &= 0, \\ c_2^2 \nabla^2 \psi - (1 - \tau_{\ell}^2 \nabla^2) \ddot{\psi} &= 0 \end{aligned}$$

where  $c_1$  and  $c_2$  are respectively, classical phase velocities defined by

$$(5.4) \quad c_1 = \left( \frac{\lambda + 2\mu}{\rho} \right)^{\frac{1}{2}}, \quad c_2 = \left( \frac{\mu}{\rho} \right)^{\frac{1}{2}}$$

We now try solutions of (5.3) in the forms of surface waves characterized by

$$\phi = A \exp[-k\nu_1 x_2 + ik(x_1 - ct)] , \quad (5.5)$$

$$\psi = B \exp[-k\nu_2 x_2 + ik(x_2 - ct)]$$

Equations (5.3) are satisfied if  $\nu_\alpha$  are given by

$$(5.6) \quad \nu_\alpha^2 = 1 - (c/c_\alpha)^2 [1 - k^2 \tau^2 \ell^2 (c/c_\alpha)^2]^{-1}, \quad \alpha=1,2 .$$

Using (5.5) in (5.2), we obtain the displacement field and carrying (5.2) into the constitutive equations (2.2) to (2.5), we arrive at the stress field.

Thus, for example, at  $x_2 = 0$  we have

$$\begin{aligned} t_{22} &= [(c_1/c_2)^2 (\nu_1^2 - 1) + 2] M_1 A + 2i\nu_2 M_2 B \\ (5.7) \quad t_{21} &= -2i\nu_1 M_1 A + (1 + \nu_2) M_2 B \end{aligned}$$

where



$$\begin{aligned}
 M_1 \equiv & \int_0^\infty dx_2' \int_{-\infty}^\infty dx_1' K_0 \{[(x_1' - x_1)^2 + x_2'^2]^{1/2}/\tau\ell\} \\
 & \cdot \exp [-k v_1 x_2' + 2k(x_1' - ct)] \\
 (5.8)
 \end{aligned}$$

$$\begin{aligned}
 M_2 \equiv & \int_0^\infty dx_2' \int_{-\infty}^\infty dx_1' K_0 \{[(x_1' - x_1)^2 + x_2'^2]^{1/2}/\tau\ell\} \\
 & \cdot \exp [-k v_2 x_2' + ik(x_1' - ct)]
 \end{aligned}$$

But  $t_{22}$  and  $t_{21}$  must vanish at  $x_2 = 0$ . Hence, we must have

$$(5.9) \quad [(c_1/c_2)^2 (v_1^2 - 1) + 2](1 + v_2^2) - 4 v_1 v_2 = 0$$

provided  $M_1 M_2 \neq 0$ . This is the Rayleigh determinant for nonlocal elastic surface waves. Upon carrying (5.6) into (5.9), eventually we rearrange (5.9) into the form

$$(5.10) \quad \gamma[a_1 \gamma^3 + a_2 \gamma^2 + a_3 \gamma + a_4] = 0$$

where

$$a_1 = \frac{1}{16} + \frac{m+1}{4} \epsilon^2 + \frac{1+4m-3m^2}{4} \epsilon^4 + m(1-m)\epsilon^6,$$

$$(5.11) \quad a_2 = -\frac{1}{2} + \frac{2m^2 - m - 3}{2} \epsilon^2 - (1 + m - 2m^2) \epsilon^4,$$

$$a_3 = \frac{3}{2} - m + (2 - m - m^2) \epsilon^2,$$

$$a_4 = 1 - m,$$

$$\gamma = (c/c_2)^2, \quad \epsilon = k\tau l, \quad m = \frac{1-2\nu}{2(1-\nu)}$$

and  $\nu$  is Poisson's ratio.

It is not difficult to verify that for  $\epsilon = 0$ , (5.10) reduces to the classical Rayleigh function whose roots are recorded in Ref. [14] for various  $\nu$ . It is also clear that the roots of (5.10) is a function of  $\epsilon$ , consequently, the Rayleigh wave velocity is dispersive. In Table 1 below, we give values of  $\sqrt{\gamma} = c_R/c_2$  as a function of  $\epsilon = e_0 ka$ , for various Poisson's ratios  $\nu$ . Phase velocities  $c_R/c_2$  versus  $\epsilon = e_0 ka$  is plotted in Fig. 4 for various values of  $\nu$ . The non-dimensional frequency  $\omega\epsilon/c_2$ , versus  $\epsilon$  are displayed in Table 2 and Fig. 5. To provide a comparison of these results with the lattice dynamic calculations, we divide the abscissa and ordinate of Fig. 4 by  $e_0 = 0.31$ . Fig. 6 shows  $ka/c_2$  versus  $ka$  of nonlocal results. Comparison with atomic lattice dynamic calculations carried out by Wallis and Gazis<sup>15</sup> for  $KCl$  is excellent. We have used  $\nu = 0.3$  to approximate the  $KCl$  molecules with an isotropic solid. The lattice dynamic calculations were made for waves propagating in the (100) direction on a (001) surface.

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TABLE I

Phase Velocity of Rayleigh Surface Waves

$\epsilon = \epsilon_0 ka$	$c_R/c_2$					
	$\nu = 0.0$	$\nu = 0.1$	$\nu = 0.2$	$\nu = 0.3$	$\nu = 0.4$	$\nu = 0.5$
0	0.87402	0.89311	0.91099	0.9274	0.9422	0.95532
0.1	0.86382	0.88418	0.90337	0.92096	0.9366	0.94998
0.2	0.8356	0.85902	0.88162	0.90242	0.92042	0.93446
0.3	0.79476	0.82176	0.84857	0.87377	0.89519	0.91025
0.4	0.74719	0.77715	0.80798	0.83775	0.86318	0.8794
0.5	0.69757	0.72944	0.76336	0.79724	0.82664	0.84415
0.6	0.64889	0.68173	0.71768	0.75479	0.78772	0.80646
0.7	0.60288	0.63598	0.67295	0.71226	0.74819	0.76809
0.8	0.56044	0.59312	0.63039	0.67105	0.7093	0.73021
0.9	0.52178	0.55374	0.5907	0.63193	0.67187	0.69363
1.0	0.4869	0.51786	0.55407	0.59537	0.63637	0.65882

TABLE II

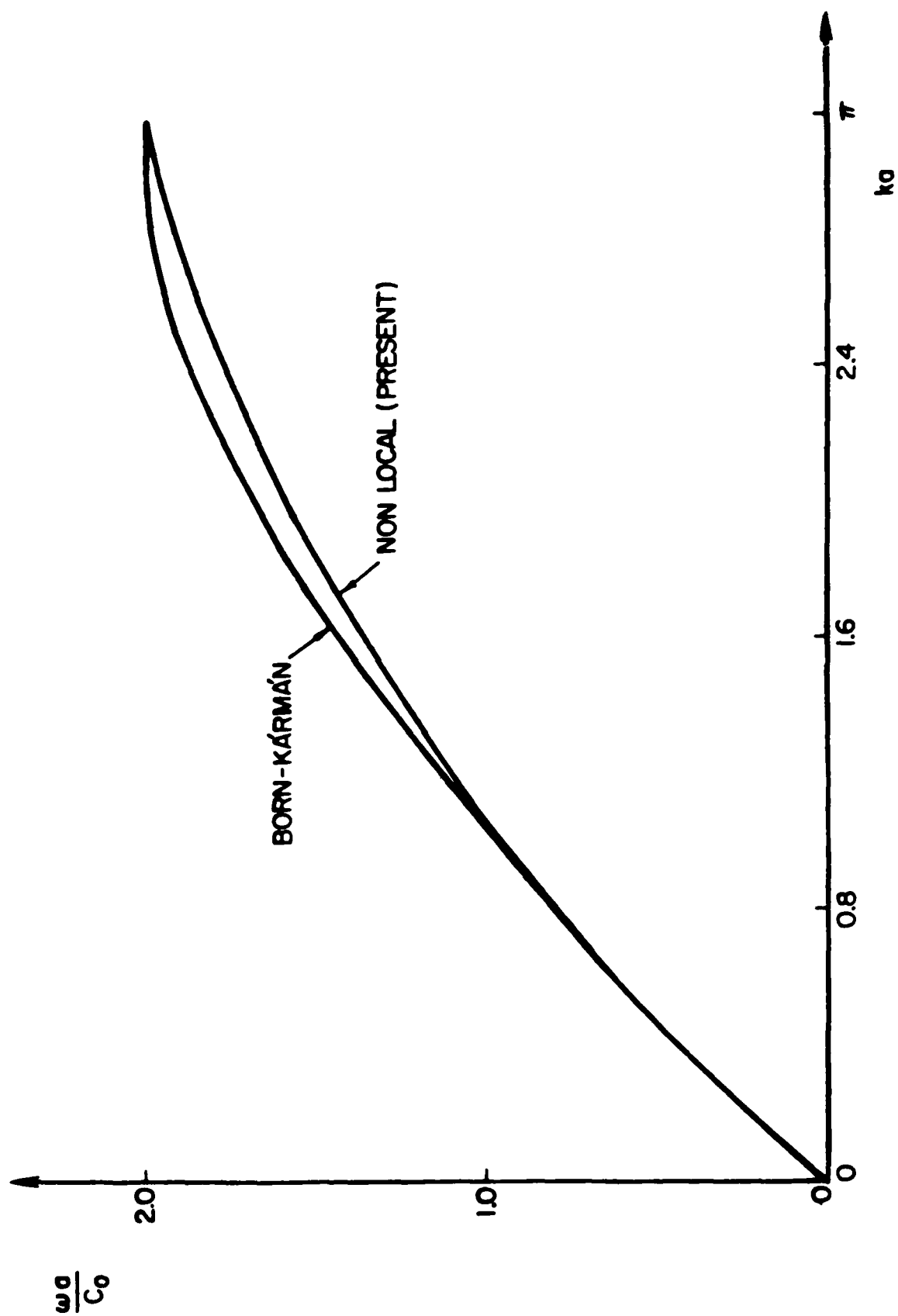
Dispersion of Rayleigh Surface Waves

$\epsilon = \epsilon_0 ka$	$\omega a/c_2$					
	$\nu = 0.0$	$\nu = 0.1$	$\nu = 0.2$	$\nu = 0.3$	$\nu = 0.4$	$\nu = 0.5$
0	0	0	0	0	0	0
0.1	0.086382	0.08842	0.09034	0.09210	0.09366	0.09500
0.2	0.16712	0.1718	0.17632	0.18048	0.18408	0.18689
0.3	0.23843	0.24653	0.25457	0.26213	0.26856	0.27307
0.4	0.29888	0.31086	0.32319	0.3351	0.34527	0.35176
0.5	0.34878	0.36472	0.38168	0.39862	0.41332	0.42207
0.6	0.38933	0.40904	0.43061	0.45288	0.47263	0.48388
0.7	0.42202	0.44518	0.47106	0.49858	0.52373	0.53766
0.8	0.44835	0.4745	0.50431	0.53684	0.56744	0.58417
0.9	0.4698	0.49837	0.53163	0.56874	0.60469	0.62427
1.0	0.4869	0.51786	0.55407	0.59537	0.63637	0.65882

## FIGURE CAPTIONS

### Figure

- |   |   |
|---|---|
| 1 | Dispersion Curves                               |
| 2 | Screw Dislocation                               |
| 3 | Non-Dimensional Hoop Stress                     |
| 4 | Phase Velocity of Rayleigh Surface Waves        |
| 5 | Dispersion of Rayleigh Surface Waves            |
| 6 | Dispersion Relations for Rayleigh Surface Waves |



DISPERSION CURVES

FIGURE 1

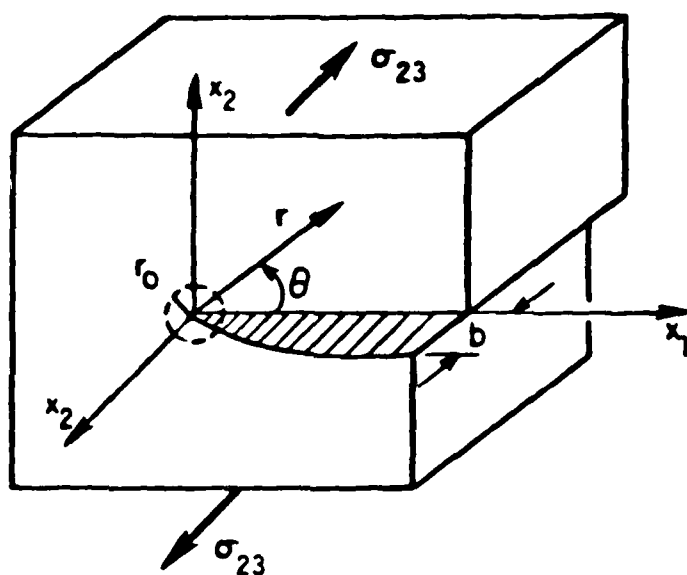
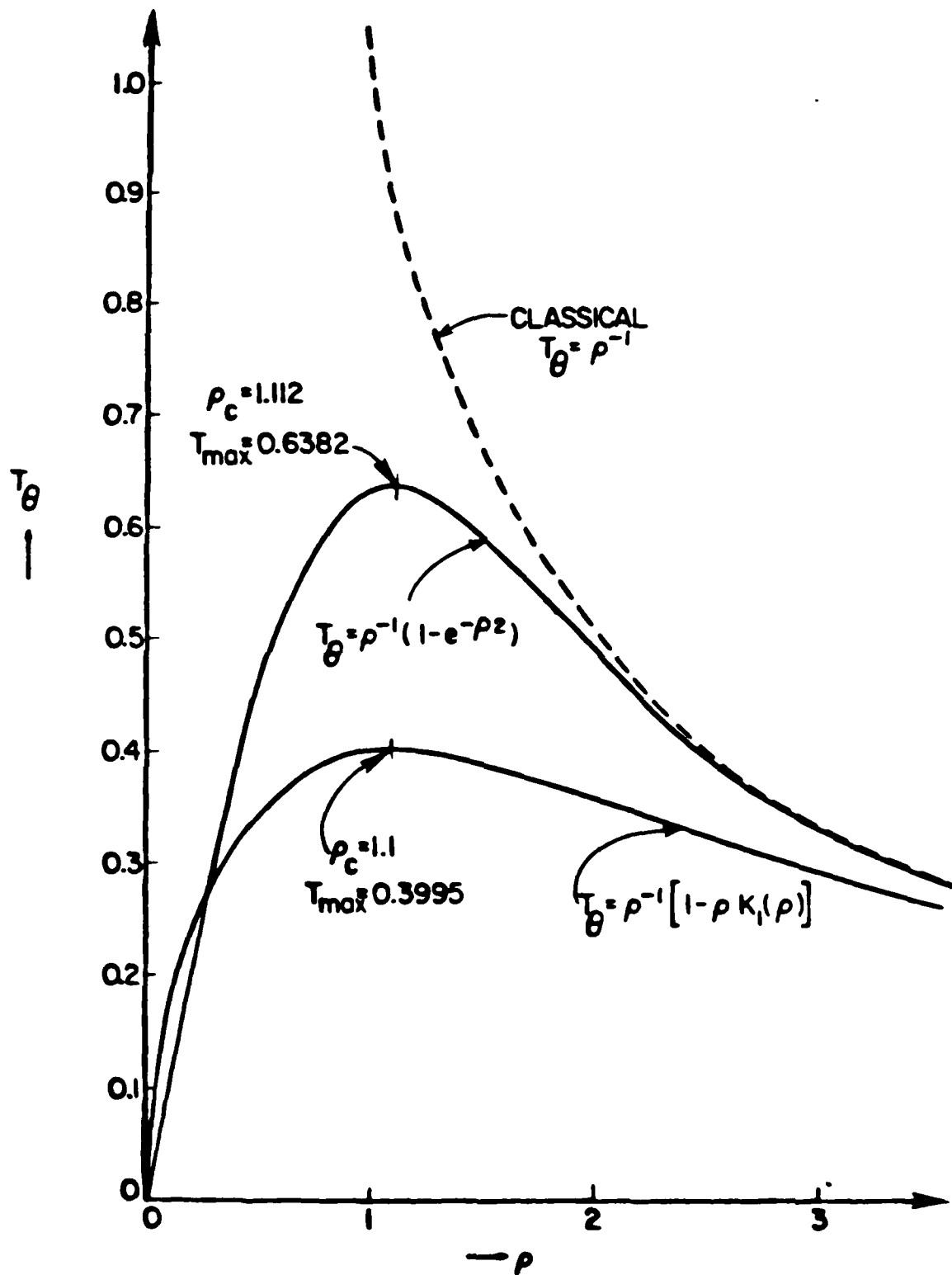
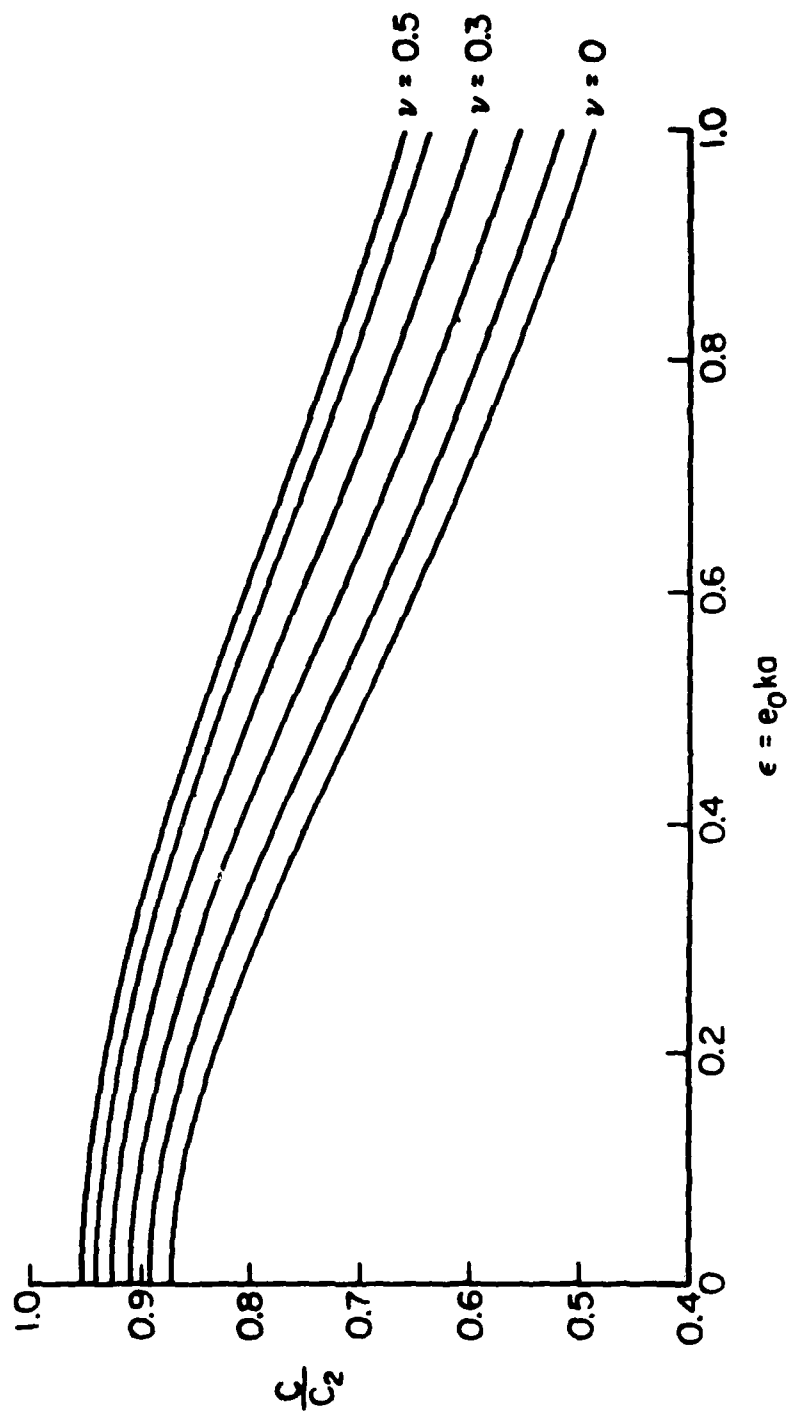


FIGURE 2 : SCREW DISLOCATION



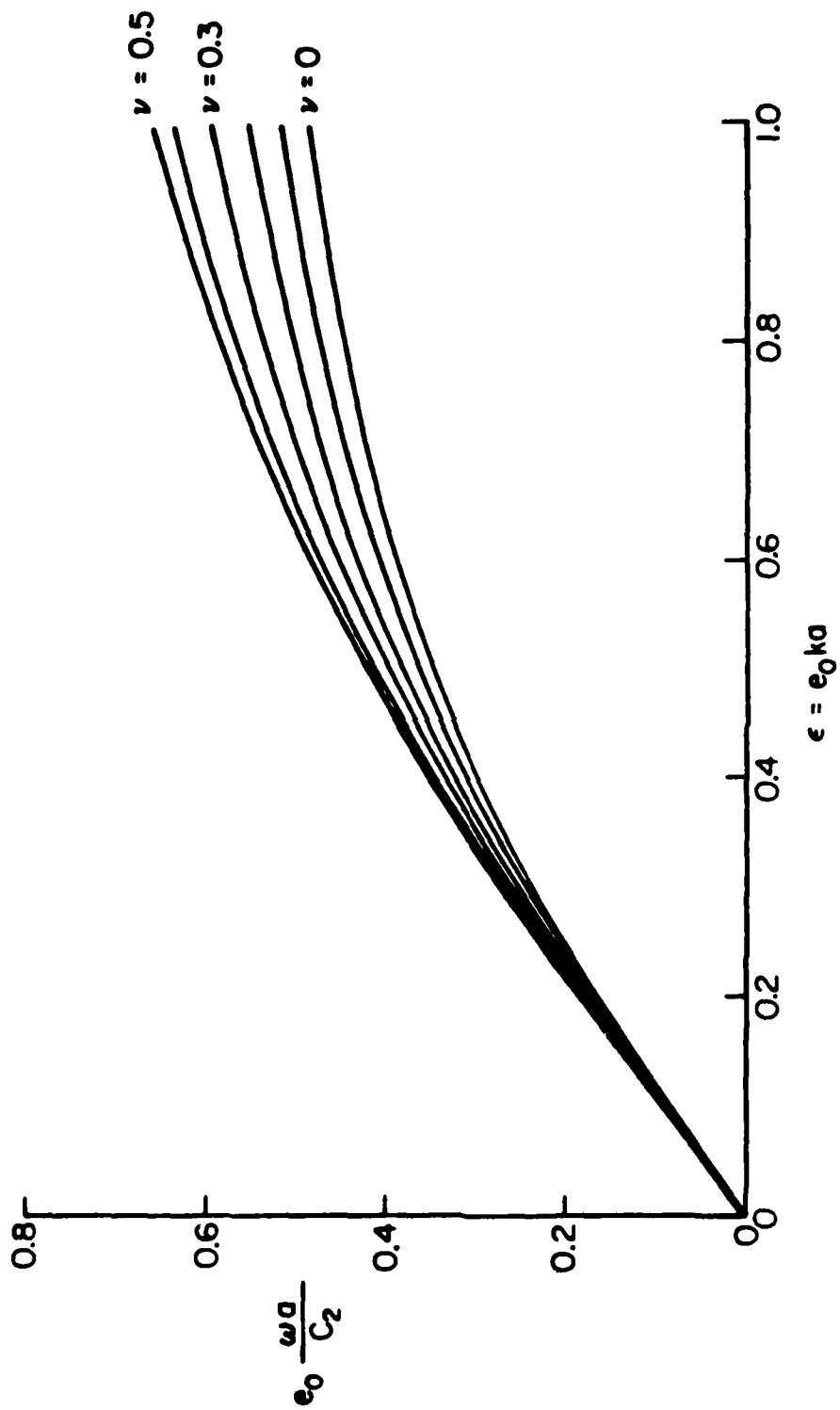


NON-DIMENSIONAL HOOP STRESS  
FIGURE 3



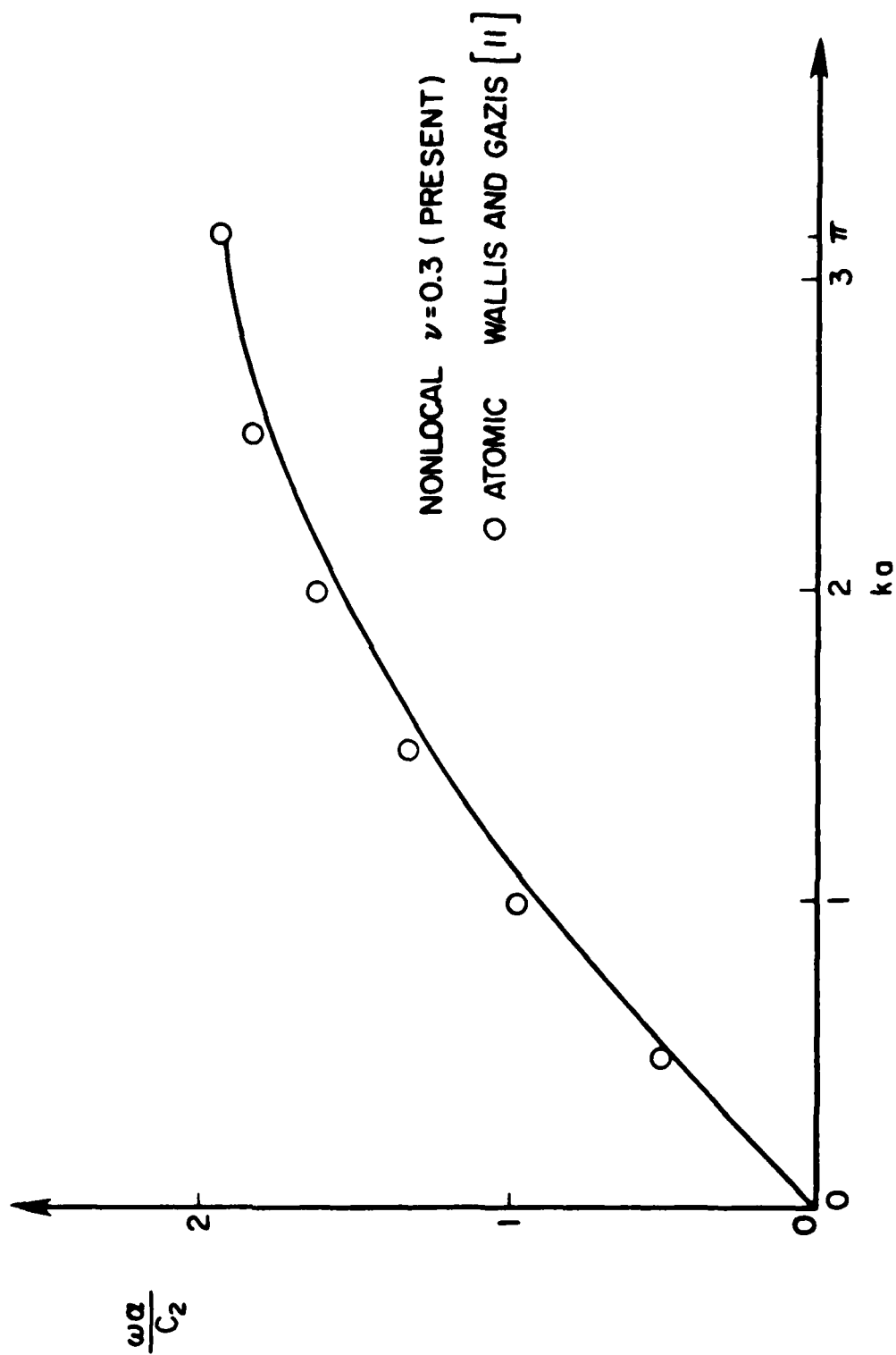
PHASE VELOCITY OF RAYLEIGH SURFACE WAVES

FIGURE 4



DISPERSION OF RAYLEIGH SURFACE WAVES

FIGURE 5



DISPERSION RELATIONS FOR RAYLEIGH SURFACE WAVES

FIGURE 6

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